Revising partial pre-orders with partial pre-orders: 
A unit-based revision framework

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Abstract

Belief revision studies strategies about how agents revise their belief states when receiving new evidence. Both in classical belief revision and in epistemic revision, a new input is either in the form of a (weighted) propositional formula or a total pre-order (where the total pre-order is considered as a whole). However, in some real-world applications, a new input can be a partial pre-order where each unit that constitutes the partial pre-order is important and should be considered individually. To address this issue, in this paper, we study how a partial pre-order representing the prior epistemic state can be revised by another partial pre-order (the new input) from a different perspective, where the revision is conducted recursively on the individual units of partial pre-orders. We propose different revision operators (rules), dubbed the extension, match, inner and outer revision operators, from different revision points of view. We also analyze several properties for these operators.

Introduction

Belief revision is a framework that characterizes the process of belief change in order to revise an agent’s current beliefs to accommodate new evidence and to reach a new consistent set of beliefs. Logic-based belief revision and epistemic state based revision have been studied extensively (Katsuno and Mendelzon 1991; Darwiche and Pearl 1997; Benferhat et al. 2000; Booth and Meyer 2006; Ma and Liu 2009; Ma, Liu, and Benferhat 2010; Ma and Liu 2011), etc.. For numerical uncertainty formalisms, Jeffrey’s rule (Jeffrey 1983) provides the most common revision strategy for revising a probability distribution with a probability measure on a partitioned set. This strategy was extended to generate its direct counterparts for revision in possibility theory (Benferhat et al. 2009), for ordinal conditional functions (Spohn 1988), and for mass functions (Ma et al. 2010; 2011).

For logic/epistemic based revision, there are few works focusing on revision of partial pre-orders on the set of possible worlds. Revision strategies in these cases are defined in terms of either a total pre-order revised by another total pre-order (e.g., (Nayak 1994; Benferhat et al. 2000)) or a partial pre-order revised by a propositional formula (e.g., (Benferhat, Lagrue, and Papini 2005)). In (Benferhat et al. 2000), the epistemic state, representing initial information, and the input, representing new information, are both total pre-orders. In (Benferhat, Lagrue, and Papini 2005), the initial epistemic state is indeed a partial pre-order, however, the input information is a propositional formula. In (Bochman 2001), different strategies have been proposed to revise an epistemic state represented by a partial pre-order on the possible worlds. However, in this book there are no revision methods for revising a partial pre-order by a partial pre-order. Our revision operations are also totally different from Lang’s works on preference (e.g. (Lang and van der Torre 2008)), and Weydert, Freund and Kern-Isberner’s revision with conditionals (e.g., (Weydert 1994; Freund 1998; Kern-Isberner 2002)).

In this paper, we investigate revision strategies for this setting: a partial pre-order revised by another partial pre-order. With this perspective, each individual ordering relation (a pair of elements with an ordering connective), which we name unit, contained in the input is itself an important piece of evidence that should be preserved (Ma, Liu, and Hunter 2011).

To propose a revision framework for partial pre-orders, we investigate how a revision operator should be designed. Generally speaking, both a priori ordering set, S, and a new input S’ can be seen as sets containing individual ordering relations, e.g., the units. So, revision can be carried out by (i) deriving maximal supersets of S’ that contain suitable units in S which do not lead to possible contradiction; (ii) by inserting units from S’ to S while removing any units that are inconsistent with this insertion; or (iii) by enlarging S’ through inserting one unit from S at a time, while maintaining consistency, etc. Based on these intuitions, we propose a family of unit-based revision operators, dubbed extension revision, match revision, inner revision, and outer revision. We prove the equivalence between these operators except the match revision operator. This result is significant.
because these revision operators start from different points of view of revision, and in some sense, they can be seen as justifications for each other.

We then prove that these revision operators satisfy some appropriate properties for partial pre-order revision.

To summarize, this paper makes the following main contributions:

- We present a general framework to deal with revision where the inputs are partial pre-orders. This framework departs from existing works on revision because we emphasize the importance of individual unit encapsulated in a partial pre-order during revision.
- Several revision strategies, dubbed the extension, match, inner and outer revision operators, are proposed to handle revision from different points of view, as detailed before.
- We prove some important properties among these revision strategies.

The remainder of the paper is organized as follows. We introduce some necessary notations and definitions in Section 2. We then propose extension based revision and insertion based revision operators in Sections 3 and Section 4, respectively. Section 5 contains properties of the proposed revision operators. Finally, we conclude this paper in Section 6.

Notations and Definitions

We use $W$ to denote a finite set of interpretations (can be any set of elements). Let $\preceq$ be a pre-order over $W$ where $w \preceq w'$ means that $w$ is at least as preferred as $w'$. Two operators, $\prec$ and $\approx$, are defined from $\preceq$ in a usual sense. Note that as $\preceq$ implies $\prec$ or $\approx$ while $\prec$ (or $\approx$) is a pure relation, in this paper, we only focus on pure relations $\prec$ and $\approx$. Each $w \prec w'$ or $w \approx w'$ is called a unit. A partial pre-order is a finite set of units. Let $S$ be a set of units, we use $Sym(S)$ to denote the set of symbols from $W$ appearing in $S$.

**Definition 1** A set of units $S$ is closed iff

- $w \prec w' \in S$ implies $w' \prec w \in S$;
- $\forall w_1, w_2, w_3 \in W$, if $w_1 \prec w_2 \in S$ and $w_2 \prec w_3 \in S$ and $w_1 \prec w_2 \wedge w_2 \prec w_3 \in S$, then $w_1 \prec w_3 \in S$, where $(w_i, \in \mathbb{R})$ is either $\approx$ or $\prec$.

A set $S$ can be extended to a unique minimal closed set based on transitivity and symmetry of $\prec$ and $\approx$. We use $Cm(S)$ to denote this unique minimal closed set extended from $S$.

**Example 1** Let $S = \{w_1 \prec w_2, w_2 \prec w_3\}$, then $Cm(S) = \{w_1 \prec w_2, w_2 \prec w_3, w_1 \prec w_3\}$.

$S$ is closed when it cannot be extended further. This is the counterpart of the deductive closure of a knowledge base $K$ under logical connectives in classical logics.

**Definition 2** A subset $C$ of $S$ is a cycle if $C = \{w_1 \prec R_1 w_2, R_2 w_3, \ldots, w_2 \prec w_3, w_1 \prec w_1\}$ s.t. $\exists R_i, R_i$ is $\prec$ for $1 \leq i \leq n$. $C$ is minimal if there does not exist a cycle $C'$ s.t. $Cm(C') \subset Cm(C)$.

If $S$ has a cycle, then $S$ is said to be inconsistent. Otherwise it is said to be consistent or free of cycles. If $S$ is closed and contains cycles, then all minimal cycles are of the form $\{a \prec b, b \prec c, c \prec a\}$ or $\{a \prec b, b \prec a\}$, i.e., only two units.

Any unit $w R w'$ is called a free unit if $w R w'$ is not involved in any cycle in $S$. The concept of free unit is the counterpart of free formula concept in logic-based inconsistency handling (Benferhat, Dubois, and Prade 1992; Hunter and Konieczny 2006).

**Example 2** Let $S = \{w_1 \prec w_2, w_2 \prec w_3, w_3 \prec w_4, w_4 \prec w_1\}$, then $C_1 = \{w_1 \prec w_2, w_2 \prec w_3, w_3 \prec w_1\}$ is a minimal cycle whilst $C_2 = \{w_1 \prec w_2, w_2 \prec w_3, w_3 \prec w_4, w_4 \prec w_1\}$ is a cycle but not minimal, since the subsequence $w_1 \prec w_3 \approx w_4, w_4 \prec w_1$ in $C_2$ can be replaced by $w_3 \prec w_4$ and hence forms $C_1$.

For any set of units $S$, we use $|S|$ to count the number of distinct units in $S$. Therefore, $w \prec w'$ and $w' \prec w$ are counted as one instead of two units in $|S|$. So for $S = \{w_1 \prec w_2, w_2 \prec w_3, w_3 \prec w_1, w_1 \prec w_2\}$, we have $|S| = 3$.

Without loss of generality, subsequently, if without other specifications, we assume that a set of units $S$ and any new input $S_I$ are both closed and free of cycles. For convenience, we use $S_{CC}$ to denote the set of all closed and consistent sets of units (free of cycles) w.r.t. a given $W$ and $\{\approx, \prec\}$. Subsequently, we will denote $\circ$ a revision operator associating a resultant set of units $\hat{S} = S \circ S_I$ with two given sets, one represents the prior state ($S$) and the other new evidence ($S_I$).

**Extension based Revision**

In this section, we define the extension based revision operator. The basic idea is to extend $S_I$ to include the units in $S$.

**Definition 3** For any $S, S' \in S_{CC}$, a consistent set of units $S''$ is called a maximal strict extension of $S'$ to $S$ if it satisfies the following conditions:

- $S' \subseteq S''$.
- If at least one of the three units $w \prec w'$, $w' \prec w$ or $w' \prec w'$ can be inferred by $S \cup S'$, then $S''$ also contains a unit connecting $w$ and $w'$.
- If none of the three units $w \prec w'$, $w' \prec w$ or $w' \prec w'$ can be inferred by $S \cup S'$, then $S''$ does not contain any unit connecting $w$ and $w'$ either.
- For any consistent set of units $S''$ that satisfies the above three conditions, $|S'' \cap S| \geq |S'' \cap S|$. The strict extension of $S'$ to $S$ is an extension of $S'$ that contains just enough units to cover all the possible units that might exist in the resultant set. From the 4th condition, we can see that $S''$ already contains as many of the units in $S$ as possible. Strict extensions do exist for any complete and consistent $S$ and $S'$.

**Proposition 1** For any $S, S' \in S_{CC}$, there exists at least one strict extension of $S'$ to $S$.

In fact, there could be several such strict extensions.

**Example 3** Let $S = \{w_1 \prec w_2, w_2 \prec w_3, w_3 \prec w_1\}$ and $S' = \{w_1 \prec w_3\}$, then there are two strict extensions of $S'$ to $S$:...
\{w_1 < w_3, w_1 \approx w_2, w_2 < w_1, w_2 < w_3\} and
\{w_1 < w_3, w_3 < w_2, w_2 < w_1\}

For any \(S, S' \in S_{CC}\), let \(\text{StrE}(S', S)\) denote the set of all strict extensions of \(S'\) to \(S\). Now we can define an extension revision operator.

**Definition 4** For any \(S, S_I \in S_{CC}\), the extension revision operator \(\circ_{ext}\) is defined as follows:

\[
S \circ_{ext} S_I = \text{Cm}(\bigcap_{S' \in \text{StrE}(S_I, S)} S').
\]  

(1)

That is, only units that can survive from all the strict extensions of \(S_I\) to \(S\) are retained after revision. Their successful survival in every strict extension shows that they do not potentially conflict with \(S_I\). This will be made clearer in later sections.

**Example 4** (Example 3 Cont.) Let \(S = \{w_1 \approx w_2, w_2 \approx w_1, w_3 < w_1, w_3 < w_2, w_1 < w_3, w_2 < w_3\}\) and \(S' = \{w_1 < w_3\}\). Note that units \(w_1 \approx w_2, w_2 \approx w_1\) are already contained in \(S\) and units \(w_3 < w_1, w_3 < w_2\) are potentially conflict with \(w_1 < w_3, w_2 < w_3\) since from \(w_2 \approx w_1, w_3 < w_2\) we can infer \(w_3 < w_1\) which contradicts with \(w_1 < w_3\). Therefore it is reasonable that they are not included in the revised set.

**Insertion Based Revision**

In this section, we introduce insertion based revision operators. The intuition here is to insert one set of units into another, and remove any units that contribute to inconsistency. However, different revision strategies can be formalized by different insertion methods. Below we propose three insertion strategies, i.e., the match, the inner and the outer revision.

**Match Revision**

The key idea of match revision is to remove any units in \(S\) which join at least one minimal cycle in \(S \cup S_I\). Therefore, these units are potentially conflicting with \(S_I\).

**Definition 5** For any \(S, S_I \in S_{CC}\), let \(S' = \text{Cm}(S \cup S_I)\) and let \(C\) be the set of all minimal cycles of \(S'\), then the match revision operator \(\circ_{match}\) is defined as:

\(S \circ_{match} S_I = \text{Cm}(S' \setminus \bigcup_{C \in C} C \setminus S_I)\).

**Example 5** Let \(S = \{w_3 < w_2, w_2 < w_4, w_4 < w_1, w_3 < w_4, w_3 < w_1, w_2 < w_3\}\) and \(S_I = \{w_1 < w_2, w_2 < w_4, w_4 < w_3\}\). then we have six minimal cycles in \(\text{Cm}(S \cup S_I)\). That is

\[
C_1 : w_1 < w_2, C_2 : w_1 < w_4, w_4 < w_1, C_3 : w_2 < w_4, w_4 < w_2, C_4 : w_3 < w_4, w_4 < w_3, C_5 : w_3 < w_4, w_4 < w_3, C_6 : w_1 < w_3, w_3 < w_1.
\]

Hence we have:

\[
S \circ_{match} S_I = \text{Cm}(\{w_1 < w_2, w_4 < w_3\}) = \{w_1 < w_2, w_4 < w_3\}.
\]

However, the match revision operator removes too many units from the prior state \(S\), as we can see from Example 5. In fact, if certain units are removed from \(S\), then there will be no cycles in \(S \cup S_I\), hence some other units subsequently could have been retained. That is, there is no need to remove all the conflicting units at once, but one after the other. This idea leads to the following inner and outer revision operators.

**Inner Revision**

The basic idea of inner revision is to insert each unit of \(S_I\) one by one into \(S\), and in the meantime, remove any unit in \(S\) that are inconsistent with the inserted unit. The motivation of the one by one insertion instead of the batch insertion in the match revision can be illustrated by the following example.

**Example 6** (Ex. 5 Cont') Let \(S\) and \(S_I\) be the same as in Ex. 5, then if we insert \(w_4 < w_3\) into \(S\), then there are three minimal cycles in \(\text{Cm}(S \cup \{w_4 < w_3\})\), i.e., \(C_1 : w_2 < w_4, w_4 < w_2, C_2 : w_3 < w_2, w_2 < w_3, C_3 : w_3 < w_4, w_4 < w_3\). Hence we can remove the units in \(C_1, C_2\) and \(C_3\) from \(\text{Cm}(S \cup \{w_4 < w_3\})\) except \(w_4 < w_3\) since it is from \(S_I\), and we obtain \(S' = \{w_3 < w_1, w_4 < w_3, w_2 < w_1, w_4 < w_1\}\). Then if we insert \(w_1 < w_2\) in \(S\) and remove the units in the minimal cycle \(w_1 < w_2, w_2 < w_1\) from \(\text{Cm}(S' \cup \{w_1 < w_2\})\), we get: \(S'' = \{w_1 < w_2, w_4 < w_3, w_3 < w_1, w_4 < w_1, w_4 < w_2, w_3 < w_2\}\) which retains more units than the match revision.

Of course, the revision result depends on the order in which these units from \(S_I\) are inserted to \(S\). Hence, only the units that exist in all revision results for any insertion order should be considered credible for the final, consistent revision result.

For a set of units \(S\), let \(\text{PMT}(S)\) denote the set of all permutations of the units in \(S\). For example, if \(S = \{w_1 < w_3, w_2 < w_3\}\), then \(\text{PMT}(S) = \{\{w_1 < w_3, w_2 < w_3\}, \{w_2 < w_3, w_1 < w_3\}\}\).

**Definition 6** \(\forall S, S_I \in S_{CC}\), let \(\overline{T} = (t_1, \ldots, t_i)\) be a permutation in \(\text{PMT}(S_I)\), then the result of sequentially inserting \(\overline{T}\) into \(S\) one by one, denoted as \(S_{\overline{T}}\), is defined as:

- Let \(S_I\) be the resultant set by sequentially inserting \(t_1, \ldots, t_i\) one by one. Let \(S' = \text{Cm}(S \cup \{t_i+1\})\) and \(C\) be the set of all minimal cycles of \(S'\), then \(S_{I+1} = S' \setminus (\bigcup_{C \in C} C \setminus S_I)\).

**\(S_{\overline{T}}\)**

The inner revision operator can be defined as follows.

**Definition 7** For any \(S, S_I \in S_{CC}\), the inner revision operator is defined as:

\[
S \circ_{in} S_I = \text{Cm}(\bigcap_{\overline{T} \in \text{PMT}(S_I)} S_{\overline{T}}).
\]  

(2)

**Example 7** (Ex. 6 Cont') Let \(S\) and \(S_I\) be the same in Ex. 5, then we have \(S_{\{w_4 < w_3, w_1 < w_2\}} = \{w_1 < w_2, w_4 < w_3, w_3 < w_1, w_4 < w_1, w_4 < w_2, w_3 < w_2\}\) and \(S_{\{w_1 < w_2, w_4 < w_3, w_3 < w_1, w_4 < w_1, w_4 < w_2, w_3 < w_2\}} = \{w_1 < w_2, w_4 < w_3, w_3 < w_1, w_4 < w_1, w_4 < w_2, w_3 < w_2\}\). Hence \(S \circ_{in} S_I = \{w_1 < w_2, w_4 < w_3, w_3 < w_1, w_1 < w_3, w_1 < w_2, w_4 < w_2, w_4 < w_1\}\).

\[\text{Note that in general, for } t \neq t', S_{\overline{T}} \neq S_{\overline{T'}}.\]

\[\text{As two equivalence relations } w \approx w' \text{ and } w' \approx w \text{ are in fact the same relation, in this and next section, this type of relations are considered as one relation (unit) and be inserted together.}\]
Example 8 Let \( S = \text{Cm}\{w_1 < w_5, w_3 < w_5, w_5 < w_6, w_6 < w_2, w_6 < w_4\} \) and \( S_I = \{w_2 < w_1, w_4 < w_3\} \). Then we have \( S_{\text{out} < w_4 < w_5 < w_2 < w_1} = \text{Cm}\{w_2 < w_1, w_4 < w_3, w_3 < w_1, w_6 < w_2\} \) whilst \( S_{\text{out} < w_4 < w_5 < w_2 < w_1} = \text{Cm}\{w_2 < w_1, w_4 < w_3, w_3 < w_1, w_6 < w_2\} \). Obviously \( S_{\text{out} < w_4 < w_5 < w_2 < w_1} \neq S_{\text{out} < w_4 < w_5 < w_2 < w_1} \).

Outer Revision

Contrary to inner revision, the basic idea of outer revision is to insert each unit of \( S \) into \( S_I \) one by one, while removing any units that are inconsistent with \( S_I \). To some extent, it is a dual to the inner revision.

Similarly, the revision result depends on the order in which the units from \( S \) are inserted into \( S_I \). Again, only the units that preserved by all the results for any insertion order should be considered credible for the final revision result.

Definition 8 For any \( S, S_I \in \text{SCC} \), let \( \overline{g} = (g_1, \cdots, g_n) \) be a permutation in \( \text{PMT}(S) \), then the result of sequentially inserting \( \overline{g} \) into \( S \) one by one, denoted as \( S_{\overline{g}} \), is defined as follows:

- Let \( S'_I \) be a resulted set by sequentially inserting \( g_1, \cdots, g_n \), one by one, into \( S_I \). Let \( S' = \text{Cm}(S'_I \cup \{t_{i+1}\}) \) and \( C \) be the set of all minimal cycles of \( S' \), then \( S'_I = S' \setminus (\bigcup_{C_{CC} \in C} \setminus S_I) \).
- \( S_{\overline{g}} = S'_I \).

Equally, the outer revision operator is defined as:

**Definition 9** For any \( S, S_I \in \text{SCC} \), the outer revision operator is defined as follows:

\[
S \odot_{\text{out}} S_I = \text{Cm}(\bigcap_{\overline{g} \in \text{PMT}(S)} S_{\overline{g}}). \tag{3}
\]

Example 9 (Example 6 Cont) Let \( S = \{w_3 < w_2, w_2 < w_4, w_4 < w_1, w_3 < w_4, w_3 < w_1\} \) and \( S_I = \{w_1 < w_2, w_2 < w_3, w_3 < w_4, w_4 < w_2, w_4 < w_1\} \). We have \( S \odot_{\text{out}} S_I = \{w_1 < w_2, w_2 < w_3, w_3 < w_4, w_4 < w_2, w_4 < w_1\} \) based on Equation (3).

**Properties**

We now present some interesting properties of these revision strategies. First, we prove the equivalence between several revision operators proposed, which reveals some insight about partial pre-order revision. This finding is significant since these revision strategies are from different perspectives and the proof of equivalence shows that these strategies are well justified.

**Theorem 1** For any \( S, S_I \in \text{SCC} \), we have \( S \odot_{\text{ext}} S_I = S \odot_{\text{in}} S_I = S \odot_{\text{out}} S_I \).

This theorem shows that the extension, inner and outer revision strategies (and their operators) in fact have the same effect after revising \( S \) with \( S_I \). Because of this, from now on, we just simply identify them all as \( \odot \) (except \( \odot_{\text{match}} \)). For the match revision \( \odot_{\text{match}} \), the following result holds.

**Proposition 2** For any \( S, S_I \in \text{SCC} \), we have \( S \odot_{\text{match}} S_I \subseteq S \odot S_I \).

This proposition reveals that the match revision strategy can sometimes remove too many units from the prior set of units \( S \).

In addition, constructively, we also give the details of the units in \( S \) that will be retained after revision.

**Proposition 3** (Irrelevance) For any \( S, S_I \in \text{SCC} \), let \( t \) be a unit in \( S \), and let \( S = S \odot S_I \). Then:

- If \( \text{Sym}(\{t\}) \cap \text{Sym}(S_I) = \emptyset \), then \( t \in S \).
- If \( t = wRw' \) such that \( w' \in \text{Sym}(S_I) \) and \( w \notin \text{Sym}(S_I) \), then if there does not exist a unit \( t' \in S \cup S_I \) such that \( t' = w'R'w' \), then \( t \in S \).
- If \( t = wRw' \) such that \( w' \in \text{Sym}(S_I) \) and \( w \notin \text{Sym}(S_I) \), then if there does not exist a relation \( t' \in S \cup S_I \) such that \( t' = w'R'w' \), then \( t \in S \).

Condition 1 means that irrelevant (and hence consistent) information is preserved. Conditions 2 and 3 both ensure that \( t \) will not be included in any cycle.

Furthermore, we can also prove that if the prior set and a new input are in total conflict, then the revision result is simply the latter.

**Proposition 4** For any \( S, S_I \in \text{SCC} \) such that \( \forall t \in S \), there exists a cycle \( C \) of \( S \cup S_I \) satisfying \( t \in C \), then \( S \odot S_I = S \odot_{\text{match}} S_I = S_I \).

**Concluding Discussions**

Although logic-based belief revision is fully studied, revision strategies for ordering information have seldom been addressed. In this paper, we investigated the issue of revising a partial pre-order by another partial pre-order. We proposed several different revision strategies, called the extension, match, inner and outer revision operators. We proved that those revision strategies except the match one produce the same revision result. These revision strategies are also proved to satisfy some intuitive and rational properties.

The convergence of the three operators, i.e., extension, inner, and outer revision, which were proposed from totally different perspectives is an important result that justifies the validity of these operators.

To regulate revision of partial pre-orders by partial pre-orders, we have also investigated some most important constraints accepted in belief revision research as follows:

**Success postulate:** The first fundamental principle of revision is to preserve new evidence. In logic-based revisions, this constraint is imposed on beliefs. That is, for a logic-based revision \( \Psi = \Phi \oplus \mu \), the success postulate reduces to \( \text{Mod}(\Psi) \subseteq \text{Mod}(\mu) \), where \( \text{Mod}(\Psi) \) and \( \text{Mod}(\mu) \) are models of formulae representing posterior beliefs and input. In our setting, \( \Psi \) and \( \mu \) represent sets of units. Hence, the success principle means that all units in the input should be themselves precisely retained. This is different from revision on total pre-orders in (Benferhat et al. 2000) that the equivalence relation between two possible worlds in the input can be detached by the prior state, since essentially (Benferhat et al. 2000) is a logic-based revision framework where the revision is more focused on the resulting belief set.

**Minimal change principle:** The issue is to define what minimal change means in ordering relation based revision.
At the first glance, it suggests including as many units in the prior state as possible when they are consistent with an input. However, what we want in the revision result from the prior state is the credible information taken from the prior state w.r.t the units in the input. Here by credible we mean it does not have any potential conflict with the relations in the input. For example, if $S = \{w_1 \prec w_2, w_2 \prec w_3, w_1 \prec w_3\}$ and $S_1 = \{w_3 \prec w_1\}$, then $w_1 \prec w_3$ is a direct conflict with $w_3 \prec w_1$. For $w_1 \not\prec w_2, w_2 \not\prec w_3$, although they are not directly conflicting with $w_3 \prec w_1$, they are potentially conflicting with $w_2 \prec w_3$ since $w_1 \prec w_2, w_2 \prec w_3, w_3 \prec w_1$ is a minimal cycle in $S \cup S_1$. Hence the revision result should not contain any of them.

In addition, based on these principles, we have developed a set of AGM-style postulates, which are satisfied by the revision operators we proposed in this paper. The idea is to keep all the AGM postulates even if our aim is to generalize the revision process to deal with a very flexible structure which is a partial pre-order. However, we consider very different components of the revision operation. Initial epistemic state is no longer a propositional formula but a set of standard concepts need to be adapted, in particular the concepts of consistency and entailment. This clearly departs from the AGM postulates, which are satisfied by the revision process to deal with a very flexible structure. Initial epistemic state is no longer a propositional formula but a set of standard concepts need to be adapted, in particular the concepts of consistency and entailment. This clearly departs from the AGM postulates.

A possible future work is to study non-transitive pre-orders and group pre-order, e.g., $p > q$ where both $p$ and $q$ can have more than one models.

References


